Chap 5 Phasor Method

5.1 Fundamental concept of phasor method

- Consider an 3rd-order RLC circuir excited by $v(t) = V\,cos(\omega t)$

  (5.1-1) $$y'''(t) + a_2 y''(t) + a_1 y'(t) + a_0 y(t) = b_2 v''(t) + b_1 v'(t) + b_0(t)$$

  where $y(t)$ is the voltage or current of a component to be measured.

- The characteristic equation is $\lambda^3 + a_2\lambda^2 + a_1\lambda + a_0 = 0$ with roots $Re(\lambda_i) < 0$, i=1,2,3, which implies the circuit is stable.

- Taking Laplace transform yields

  (5.1-2) $$\hat{y}(s) = \underbrace{\frac{p_2 s^2 + p_1 s + p_0}{s^3 + a_2 s^2 + a_1 s + a_0}}_{\hat{p}(s)} + \underbrace{\frac{b_2 s^2 + b_1 s + b_0}{s^3 + a_2 s^2 + a_1 s + a_0}}_{\hat{h}(s)}\hat{v}(s)$$

  where $\hat{v}(s) = \dfrac{Vs}{s^2 + \omega^2}$, $\hat{p}(s)$ is related to the initial conditions and $\hat{h}(s)$ is the transfer function of the circuit.

- Hence,

  (5.1-3) $$y(t) = p(t) + h(t) * v(t) \Rightarrow y(t) = h(t) * v(t), \text{ as } t \to \infty$$

  where $p(\infty) = 0$ because $Re(\lambda_i) < 0$, i=1,2,3.

- That means $y(t)\big|_{t\to\infty}$ can be solved by

  (5.1-4) $$\hat{y}(s) = \hat{h}(s)\frac{Vs}{s^2+\omega^2} = \frac{b_2 s^2 + b_1 s + b_0}{s^3 + a_2 s^2 + a_1 s + a_0}\cdot\frac{Vs}{s^2+\omega^2}$$

  $$= \frac{q_2 s^2 + q_1 s + q_0}{s^3 + a_2 s^2 + a_1 s + a_0} + \frac{As + B\omega}{s^2+\omega^2} = \hat{q}(s) + \frac{As + B\omega}{s^2+\omega^2}$$

  which leads to $y(t) = q(t) + A\,cos\,\omega t + B\,sin\,\omega t$. Similarly, $Re(\lambda_i) < 0$, i=1,2,3, results in $q(\infty) = 0$ and

  (5.1-5) $$y(t) = A\,cos\,\omega t + B\,sin\,\omega t = Y\,cos(\omega t + \theta), \quad \text{as } t \to \infty$$

  where $Y = \sqrt{A^2 + B^2}$ and $\theta = -tan^{-1}(B/A)$.

- From (5.1-4), it can be obtained that

(5.1-6) $\hat{q}(s)(s^2+\omega^2)+(As+B\omega)=Vs\cdot\hat{h}(s)$

Let $s=j\omega$, then $s^2+\omega^2=0$ and $(As+B\omega)=Vs\cdot\hat{h}(s)$, i.e.,

(5.1-7) $(jA\omega+B\omega)=jV\omega\cdot\hat{h}(j\omega)$

$$\Rightarrow(A-jB)=V\cdot\hat{h}(j\omega)\Rightarrow Ye^{j\theta}=V\left|\hat{h}(j\omega)\right|e^{j\angle\hat{h}(j\omega)}$$

where $\hat{h}(j\omega)=\left|\hat{h}(j\omega)\right|e^{j\angle\hat{h}(j\omega)}$, $Y=V\left|\hat{h}(j\omega)\right|$ and $\theta=\angle\hat{h}(j\omega)$, then

(5.1-8) $y(t)=V\left|\hat{h}(j\omega)\right|cos\left(\omega t+\angle\hat{h}(j\omega)\right)$

- To sum up, if the input of an RLC circuit is $v(t)=V\,cos(\omega t)$, then the output is $y(t)=V\left|\hat{h}(j\omega)\right|cos\left(\omega t+\angle\hat{h}(j\omega)\right)$ which can be solved by using the Laplace tranform and setting $s=j\omega$.

5.2 Phasor of sinusoidal signals

- A signal $v(t)$ with a single frequency ω is expressed as

(5.2-1) $v(t)=V\,cos(\omega t+\theta)$

where V is the magnitude and θ is the phase of the signal.

- Using the Euler formula $e^{j\theta}=cos\,\theta+j\,sin\,\theta$ can obtain

(5.2-2) $v(t)=V\,cos(\omega t+\theta)=Re\left(Ve^{j(\omega t+\theta)}\right)=Re\left(Ve^{j\theta}e^{j\omega t}\right)=Re\left(\mathbb{V}e^{j\omega t}\right)$

where

(5.2-3) $\mathbb{V}=Ve^{j\theta}=V\angle\theta=V\,cos\,\theta+jV\,sin\,\theta$

and $\mathbb{V}=Ve^{j\theta}$ is called the phasor of $v(t)=V\,cos(\omega t+\theta)$.

Example: If $v(t)=2\,sin(4t-30°)$, determine its phasor.

Example: Determine the sinusoidal signal $v(t)$ with frequency $\omega=3$ and phasor $\mathbb{V}=5\angle 60°$.

5.3 Components in Phasor Method

- Voltage source $v_s(t) = V_s \cos(\omega t + \theta_s)$

  Since $v_s(t) = V_s \cos(\omega t + \theta_s) = Re\left(V_s e^{j\theta_s} e^{j\omega t}\right)$, the voltage source in phasor method is

  (5.3-1) $\quad \mathbf{V}_s = V_s e^{j\theta_s} = V_s \angle \theta_s$

- Current source $i_s(t) = I_s \cos(\omega t + \theta_s)$

  Since $i_s(t) = I_s \cos(\omega t + \theta_s) = Re\left(I_s e^{j\theta_s} e^{j\omega t}\right)$, the current source in phasor method is

  (5.3-2) $\quad \mathbf{I}_s = I_s e^{j\theta} = I_s \angle \theta$

- Resistor with component equation $v_R(t) = R \cdot i_R(t)$

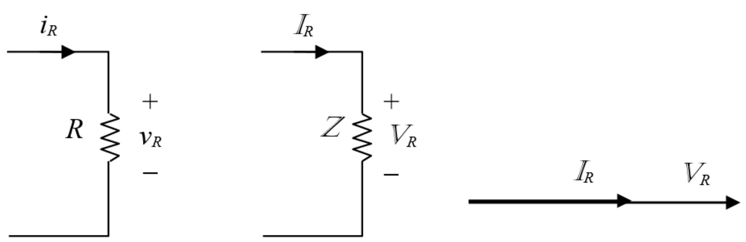
  If $v_R(t) = V_R \cos \omega t = Re\left(V_R e^{j0^\circ} e^{j\omega t}\right)$, then $i_R(t) = I_R \cos \omega t = Re\left(I_R e^{j0^\circ} e^{j\omega t}\right)$

  where $I_R = \dfrac{V_R}{R}$. Their phasors are

  (5.3-3) $\quad \mathbf{V}_R = V_R e^{j0^\circ} = V_R \angle 0^\circ$ and $\mathbf{I}_R = \dfrac{V_R}{R} e^{j0^\circ} = I_R \angle 0^\circ$.

  The impedance and admittance (or complex resistance and conductance) of a resistor are respectively defined as

  (5-3.4) $\quad \mathbf{Z}_R = \dfrac{\mathbf{V}_R}{\mathbf{I}_R} = R \; (\Omega)$ and $\mathbf{Y}_R = \dfrac{\mathbf{I}_R}{\mathbf{V}_R} = \dfrac{1}{R} \; (S)$



- Capacitor with component equation $i_C(t) = C\dfrac{dv_C(t)}{dt}$

If $v_C(t) = V_C \cos\omega t = Re\left(V_C e^{j0^\circ} e^{j\omega t}\right)$, then

$$\begin{aligned} i_C(t) &= -\omega C V_C \sin\omega t = \omega C V_C \cos\left(\omega t + 90^\circ\right) \\ &= I_C \cos\left(\omega t + 90^\circ\right) = Re\left(I_C e^{j90^\circ} e^{j\omega t}\right) \end{aligned}.$$

Their phasors are

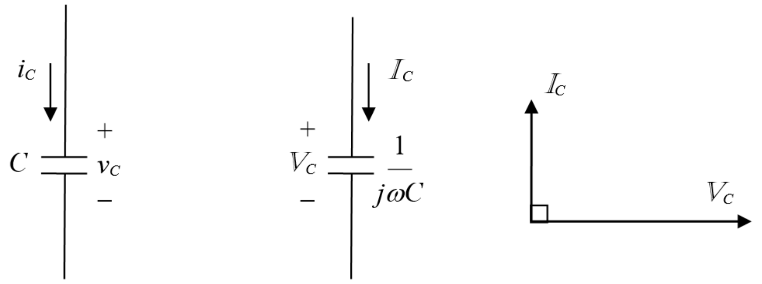
(5-3.5) $\mathbf{V}_C = V_C e^{j0^\circ} = V_C \angle 0^\circ$ and $\mathbf{I}_C = \omega C V_C e^{j90^\circ} = I_C \angle 90^\circ$.

The impedance and admittance of a capacitor are respectively defined as

(5-3.6) $\mathbf{Z}_C = \dfrac{\mathbf{V}_C}{\mathbf{I}_C} = \dfrac{V_C}{\omega C V_C \angle 90^\circ} = \dfrac{1}{\omega C}\angle(-90^\circ) = \dfrac{1}{j\omega C}$ (Ω)

(5-3.7) $\mathbf{Y}_C = \dfrac{\mathbf{I}_C}{\mathbf{V}_C} = \omega C \angle 90^\circ = j\omega C$ (S)

Compared to the voltage, the current phasor is leading 90° in phase.



- Inductor with component equation $v_L(t) = L\dfrac{di_L(t)}{dt}$

If $i_L(t) = I_L \cos\omega t = Re\left(I_L e^{j0^\circ} e^{j\omega t}\right)$, then

$$\begin{aligned} v_L(t) &= -\omega L I_L \sin\omega t = \omega L I_L \cos\left(\omega t + 90^\circ\right) \\ &= V_L \cos\left(\omega t + 90^\circ\right) = Re\left(V_L e^{j90^\circ} e^{j\omega t}\right) \end{aligned}.$$

Their phasors are

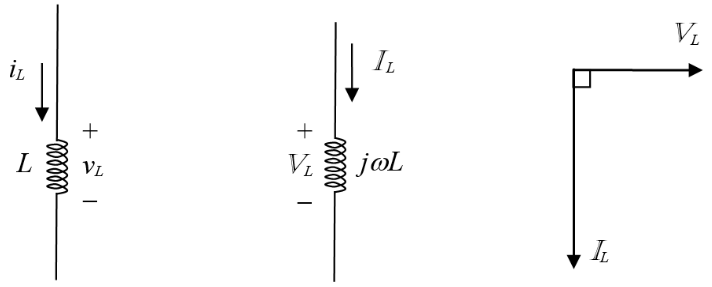
(5.3-8) $\mathbf{I}_L = I_L e^{j0^\circ} = I_L \angle 0^\circ$ and $\mathbf{V}_L = \omega L I_L e^{j90^\circ} = V_L \angle 90^\circ$.

The impedance and admittance of an inductor are respectively as

(5-3.9) $\mathbf{Z}_L = \dfrac{\mathbf{V}_L}{\mathbf{I}_L} = \dfrac{\omega L I_L \angle 90^\circ}{I_L} = \omega L \angle 90^\circ = j\omega L$ (Ω)

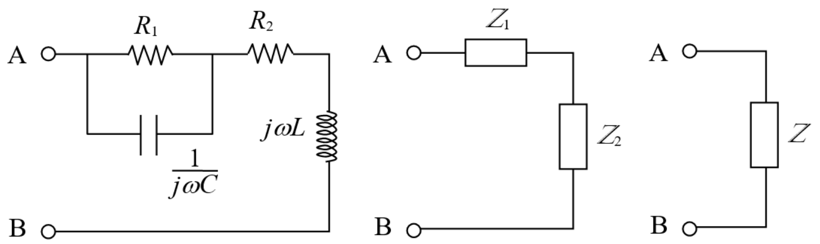
(5-3.10) $$Y_L = \frac{I_L}{V_L} = \frac{I_L}{\omega L I_L \angle 90^\circ} = \frac{1}{\omega L}\angle(-90^\circ) = \frac{1}{j\omega L} \quad \text{(S)}$$

Compared to the current, the voltage phasor is leading 90° in phase.



5.4 Circuit Analysis

- Equivalent impedance



The impedances are $Z_{R1} = R_1$, $Z_{R2} = R_2$, $Z_C = \frac{1}{j\omega C}$ and $Z_L = j\omega L$.

Since R_1 and C are in parallel and R_2 and L are in series, we have

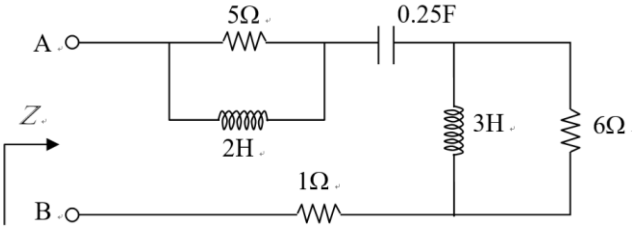
(5.4-1) $$Z_1 = \left(\frac{1}{Z_{R1}} + \frac{1}{Z_C}\right)^{-1} = \left(\frac{1}{R_1} + j\omega C\right)^{-1} = \frac{R_1}{1 + j\omega R_1 C}$$

(5.4-2) $$Z_2 = Z_{R1} + Z_L = R_2 + j\omega L$$

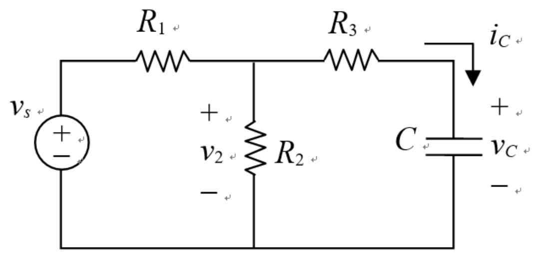
Both Z_1 and Z_2 are in series and result in the equivalent impedance

(5.4-3) $$Z = Z_1 + Z_2 = \frac{R_1}{1 + j\omega R_1 C} + R_2 + j\omega L$$

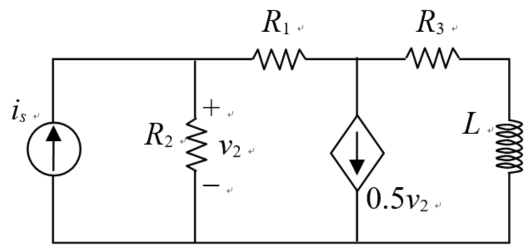
- Example: $Z = ?$



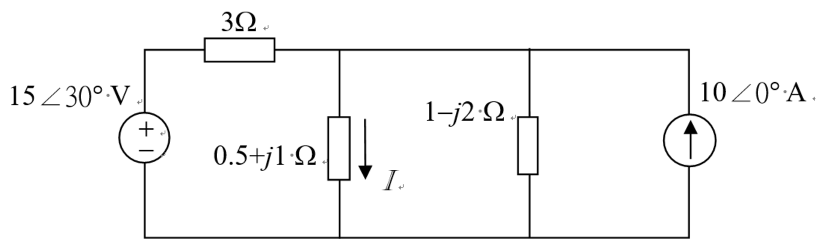
- Example: $v_2(t) = ?$



- Example: $v_2(t) = ?$

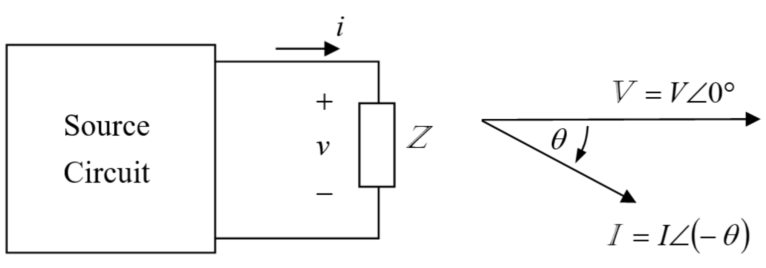


- Example: $i(t) = ?$



5.5 Complex power

- Consider a payload with impedance Z connected to a source circuit, where $v(t) = V\cos\omega t$ and $i(t) = I\cos(\omega t - \theta)$.



The phasors of $v(t) = V\cos\omega t$ and $i(t) = I\cos(\omega t - \theta)$ are $V = V\angle 0°$ and $I = I\angle(-\theta)$. The phase of the current is lagging the voltage by θ.

- The instantaneous power at t absorbed by the payload is

(5.5-1) $$p(t)=v(t)i(t)=VI\cos\omega t\cos(\omega t-\theta)$$

$$=\frac{1}{2}VI\cos\theta+\frac{1}{2}VI\cos(2\omega t-\theta)$$

$$=\underbrace{\frac{1}{2}VI\cos\theta}_{P}(1+\cos 2\omega t)+\underbrace{\frac{1}{2}VI\sin\theta}_{Q}\sin 2\omega t$$

$$=2P\cos^2\omega t+Q\sin 2\omega t$$

- The average of power in one period T is

(5.5-2) $$P_{av}=\frac{1}{T}\int_T p(t)\ dt=\frac{2P}{T}\underbrace{\int_T \cos^2\omega t\ dt}_{=T/2}+\frac{Q}{T}\underbrace{\int_T \sin 2\omega t\ dt}_{=0}$$

$$=\frac{2P}{T}\cdot\frac{T}{2}=P=\frac{1}{2}VI\cos\theta$$

Since only P is absorbed in one period, we call P the real power. On the other hand, Q is reserved in the circuit and called the reactive power.

- The root-mean-square(rms) values or effective values of $v(t)=V\cos\omega t$ and $i(t)=I\cos(\omega t-\theta)$ are calculated as

(5.5-3) $$\tilde{V}=\sqrt{\frac{1}{T}\int_T v^2(t)dt}=\sqrt{\frac{V^2}{T}\int_T \cos^2\omega t\ dt}=\sqrt{\frac{V^2}{T}\cdot\frac{T}{2}}=\frac{V}{\sqrt{2}}$$

(5.5-4) $$\tilde{I}=\sqrt{\frac{1}{T}\int_T i^2(t)dt}=\sqrt{\frac{I^2}{T}\int_T \cos^2(\omega t-\theta)\ dt}=\sqrt{\frac{I^2}{T}\cdot\frac{T}{2}}=\frac{I}{\sqrt{2}}$$

- The phasors of $v(t)=V\cos\omega t$ and $i(t)=I\cos(\omega t-\theta)$ are defined as

(5.5-5) $$\tilde{\mathbb{V}}=\frac{\mathbb{V}}{\sqrt{2}}=\frac{V}{\sqrt{2}}\angle 0^\circ=\tilde{V}\angle 0^\circ$$

(5.5-6) $$\tilde{\mathbb{I}}=\frac{\mathbb{I}}{\sqrt{2}}=\frac{I}{\sqrt{2}}\angle(-\theta)=\tilde{I}\angle(-\theta)$$

- The complex power is defined as

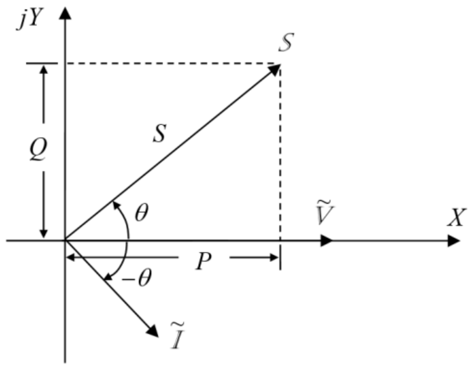
(5.5-7) $$\mathbb{S}=\tilde{\mathbb{V}}\tilde{\mathbb{I}}^*=\tilde{V}\tilde{I}\angle\theta=\frac{V}{\sqrt{2}}\frac{I}{\sqrt{2}}e^{j\theta}$$

$$=\frac{1}{2}VI\cos\theta+j\frac{1}{2}VI\sin\theta=P+jQ=S\angle\theta \quad \text{(VA: volt-ampere)}$$

where $S = |\mathbb{S}| = \tilde{V}\,\tilde{I} = \frac{1}{2}VI$ (VA) is called the apparent power. Hence,

(5.5-8) $$P = \frac{1}{2}VI\cos\theta = Re\left(\tilde{V}\ \tilde{I}^*\right) = Re\left(\mathbb{S}\right) = S\cos\theta \quad \text{(W)}$$

(5.5-9) $$Q = \frac{1}{2}VI\sin\theta = Im\left(\tilde{V}\ \tilde{I}^*\right) = Im\left(\mathbb{S}\right) = S\sin\theta \quad \text{(VAR)}$$



- Let $\mathbb{Z} = R + jX$ where *R* is the resistance and *X* is the reactance.

  Since $\tilde{V} = \tilde{I}\mathbb{Z}$, we have

(5.5-10) $$\mathbb{Z} = \frac{\tilde{\mathbb{V}}}{\tilde{\mathbb{I}}} = \frac{\tilde{V}}{\tilde{I}}\angle\theta = \frac{V}{I}\angle\theta = Z\angle\theta$$

Therefore, the complex power can be described as

(5.5-11) $$\mathbb{S} = \tilde{V}\tilde{I}^* = \tilde{I}\tilde{I}^*\mathbb{Z} = \begin{cases} \tilde{I}^2 Z\angle\theta = S\angle\theta \\ \tilde{I}^2\left(R + jX\right) = \tilde{I}^2 R + j\tilde{I}^2 X = P + jQ \end{cases}$$

where

(5.5-12) $$S = \tilde{I}^2 Z = \frac{1}{2}I^2 Z \quad \text{(VA)}$$

(5.5-13) $$P = \tilde{I}^2 R = \frac{1}{2}I^2 R \quad \text{(W)}$$

(5.5-14) $$Q = \tilde{I}^2 X = \frac{1}{2}I^2 X \quad \text{(VAR)}$$

5.6 Conservation of energy

- The conservation of energy implies the total power in a circuit is zero, i.e.,

(5.6-1) $$p(t) = \sum_{k=1}^{n} p_k(t) = \sum_{k=1}^{n} v_k(t)\,i_k(t) = 0$$

Since the power of the k^{th} component is

(5.6-2) $$p_k(t) = 2P_k \cos^2 \omega t + Q_k \sin 2\omega t, \quad k=1,2,\cdots,n$$

it can be obtained that

(5.6-3) $$\sum_{k=1}^{n} p_k(t) = \sum_{k=1}^{n}\left(2P_k \cos^2 \omega t + Q_k \sin 2\omega t\right)$$
$$= 2\left(\sum_{k=1}^{n} P_k\right)\cos^2 \omega t + \left(\sum_{k=1}^{n} Q_k\right)\sin 2\omega t = 0$$

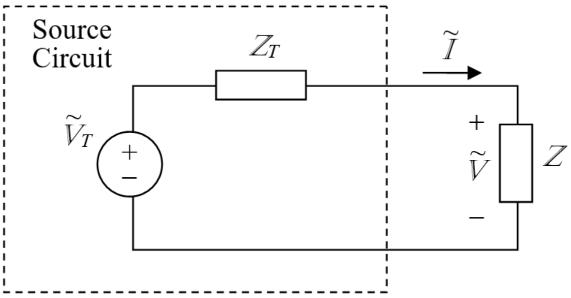
which leads to $\sum_{k=1}^{n} P_k = 0$ and $\sum_{k=1}^{n} Q_k = 0$.

Thus, the total complex power satisfies

(5.6-4) $$\mathbb{S} = \sum_{k=1}^{n} \mathbb{S}_k = \sum_{k=1}^{n} P_k + j\sum_{k=1}^{n} Q_k = 0$$

In other words, the conservation of energy also implies the total complex power is equal to zero.

5.7 The maximum power transfer theorem



- Assume $\tilde{V}_T = \tilde{V}_T\angle 0°$, $\mathbb{Z}_T = R_T + jX_T$ and $\mathbb{Z} = R + jX$, then

(5.7-1) $$\tilde{V} = \frac{\mathbb{Z}}{\mathbb{Z}_T + \mathbb{Z}}\tilde{V}_T \quad \text{and} \quad \tilde{I} = \frac{\tilde{V}_T}{\mathbb{Z}_T + \mathbb{Z}}$$

The complex power of payload is

(5.7-2) $$\tilde{V}\tilde{I}^* = \frac{\mathbb{Z}}{(\mathbb{Z}_T + \mathbb{Z})(\mathbb{Z}_T^* + \mathbb{Z}^*)}\tilde{V}_T\tilde{V}_T^*$$
$$= \frac{R + jX}{(R_T + R)^2 + (X_T + X)^2}\tilde{V}_T^2 = P + jQ$$

- The real power transferred to the payload is

(5.7-3) $$P = Re\left(\tilde{V}\tilde{I}^*\right) = \frac{R\tilde{V}_T^2}{\left(R_T + R\right)^2 + \left(X_T + X\right)^2}$$

- The maximum real power can be obtained when $\frac{\partial P}{\partial R} = 0$ and $\frac{\partial P}{\partial X} = 0$, i.e.,

(5.7-4) $$\frac{\partial P}{\partial X} = \frac{-2R\left(X_T + X\right)}{\left(\left(R_T + R\right)^2 + \left(X_T + X\right)^2\right)^2}\tilde{V}_T^2 = 0 \Rightarrow X = -X_T$$

(5.7-5) $$\frac{\partial P}{\partial R} = \frac{R_T^2 - R^2 + \left(X_T + X\right)^2}{\left(\left(R_T + R\right)^2 + \left(X_T + X\right)^2\right)^2}\tilde{V}_T^2 = 0 \Rightarrow R = R_T$$

which results in $Z = R_T - jX_T = Z_T^*$, called the matching impeadance.

- When $Z = Z_T^*$, the source circuit generates the real power

(5.7-6) $$P_T = Re\left(\tilde{V}_T\tilde{I}^*\right) = Re\left(\tilde{V}_T\left(\frac{\tilde{V}_T}{Z_T + Z}\right)^*\right) = \frac{\tilde{V}_T^2}{2R_T}$$

and the payload absorbs the maximum real power

(5.7-7) $$P_{max} = Re\left(\tilde{V}\tilde{I}^*\right) = \frac{\tilde{V}_T^2}{4R_T} = \frac{1}{2}P_T$$

That means the maximum real power can be transferred to the payload is only half of the real power generated by the source circuit.

- Consider a special case that the payload is a real resistance, i.e., $Z = R$.
Then, the real power transferred to the payload is

(5.7-8) $$P = Re\left(\tilde{V}\tilde{I}^*\right) = \frac{R}{\left(R_T + R\right)^2 + X_T^2}\tilde{V}_T^2$$

The maximum real power can be obtained when $\frac{\partial P}{\partial R} = 0$, i.e.,

(5.7-9) $$\frac{\partial P}{\partial R} = \frac{R_T^2 + X_T^2 - R^2}{\left(\left(R_T + R\right)^2 + X_T^2\right)^2}\tilde{V}_T^2 = 0 \Rightarrow R = \sqrt{R_T^2 + X_T^2}$$

- When $R = \sqrt{R_T^2 + X_T^2}$, the source circuit generates the real power

(5.7-10) $$P_T = Re\left(\tilde{V}_T\tilde{I}^*\right) = Re\left(\tilde{V}_T\left(\frac{\tilde{V}_T}{Z_T+R}\right)^*\right) = Re\left(\frac{\tilde{V}_T^2}{R+R_T-jX_T}\right)$$

$$= \frac{\tilde{V}_T^2\left(R+R_T\right)}{\left(R+R_T\right)^2+X_T^2} = \frac{\tilde{V}_T^2\left(R+R_T\right)}{R^2+2RR_T+R_T^2+X_T^2} = \frac{\tilde{V}_T^2\left(R+R_T\right)}{2R^2+2RR_T} = \frac{\tilde{V}_T^2}{2R}$$

and the payload absorbs the maximum real power

(5.7-11) $$P_{max} = Re\left(\tilde{V}\tilde{I}^*\right) = Re\left(\frac{\tilde{V}\tilde{V}^*}{R}\right) = \frac{\tilde{V}\tilde{V}^*}{R}$$

$$= \frac{R\tilde{V}_T\tilde{V}_T^*}{\left(Z_T+R\right)\left(Z_T^*+R\right)} = \frac{\tilde{V}_T^2}{2\left(R+R_T\right)} = \frac{R}{R+R_T}P_T$$

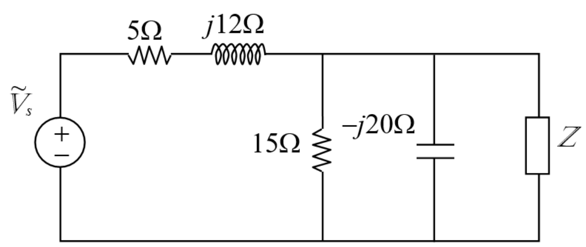
Hence, the maximum real power can be transferred to the payload is $\frac{R}{R+R_T}$ times of the real power generated by the source circuit.

- Example:

  Assume the voltage source is $\tilde{V}_s = 110\angle 0°$ V.

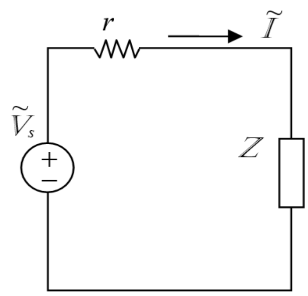
  (A) Determine the maximum real power P_{max} transferred to $Z = R + jX$.

  (B) Determine the maximum real power P_{max} transferred to $Z = R$.



5.8 Power factor correction

- The electric power transmitted from $\tilde{V}_s$ to the inductive payload $Z = R + jX$ with $X > 0$, and assume r is a small resistance of the transmission line.



- If $\tilde{V}_s = V_s\angle 0^{\circ}$, then the current is

(5.8-1) $$\tilde{I} = \frac{\tilde{V}_s}{r+Z} = \frac{\tilde{V}_s}{r+R+jX}$$

and the complex power transferred to the payload is

(5.8-2) $$S_Z = Z\tilde{I}\tilde{I}^* = \frac{(R+jX)\tilde{V}_s\tilde{V}_s^*}{(r+R)^2+X^2} = \frac{(R+jX)\tilde{V}_s^2}{(r+R)^2+X^2}$$

$$= \underbrace{\frac{R\tilde{V}_s^2}{(r+R)^2+X^2}}_{P} + j\underbrace{\frac{X\tilde{V}_s^2}{(r+R)^2+X^2}}_{Q}$$

$$= S_z\angle\theta = S_z\cos\theta + jS_z\sin\theta$$

- Define the power factor of Z as

(5.8-3) $$pf = \cos\theta = \frac{P}{S_Z} = \frac{R}{\sqrt{R^2+X^2}}$$

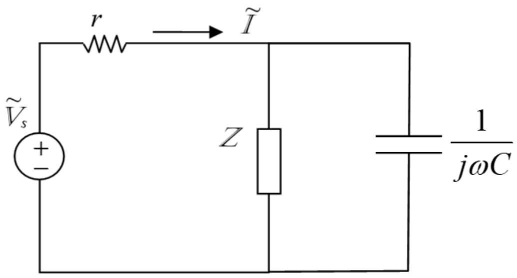
In other words, the power factor is the ratio of real power to the apparent power. That implies the power factor will be incresead if the reactance *X* is reduced.

- The power dissipated by the resistance *r* is

(5.8-4) $$P_r = r\tilde{I}\tilde{I}^* = \frac{r\tilde{V}_s\tilde{V}_s^*}{(r+R)^2+X^2} = \frac{r\tilde{V}_s^2}{(r+R)^2+X^2}$$

which implies P_r will be decreased if the reactance *X* is reduced.

- Power factor correction of inductive payload can be achieved by connecting a capacitor parallel to the payload, depicted as below:



Since $Y = Z^{-1} = \frac{1}{R+jX} = \frac{R}{R^2+X^2} - j\frac{X}{R^2+X^2}$, the new payload is

(5.8-5) $$Y_{new} = Y \,||\, j\omega C = \frac{R}{R^2+X^2} - j\frac{X}{R^2+X^2} + j\omega C$$

If $C = \dfrac{X}{\omega\left(R^2 + X^2\right)}$, then $Y_{new} = \dfrac{R}{R^2 + X^2}$, i.e.,

(5.8-6) $$Z_{new} = R + \frac{X^2}{R}$$

Hence, the power dissipated by the resistance *r* is obtained as

(5.8-7) $$P_r = r\tilde{I}\tilde{I}^* = \frac{r\tilde{V}_s\tilde{V}_s^*}{\left(r + R + \dfrac{X^2}{R}\right)^2} = \frac{r\tilde{V}_s^2}{\left(r + R + \dfrac{X^2}{R}\right)^2}$$

$$= \frac{r\tilde{V}_s^2}{\left(r+R\right)^2 + 2\left(1 + \dfrac{r}{R}\right)X^2 + \dfrac{X^4}{R^2}} < \frac{r\tilde{V}_s^2}{\left(r+R\right)^2 + X^2}$$

It is clear that the power P_r is indeed reduced after power factor correction.